SYDNEY BOYS HIGH SCHOOL

NESA Number:

Name:

Maths Class: Circle

12P 12L 12U

12Z2 12Z4

11A 11B 11M1 11M2

2023

YEAR 12
TASK 4
TRIAL HSC

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- All answers, unless otherwise stated, should be left in simplified, exact form
- Marks may **NOT** be awarded for messy or badly arranged work
- For questions in Section II, show ALL relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 2 – 5)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6 – 42)
- Attempt all Questions in Section II
- Allow about 2 hours and 45 minutes for this section

Examiner: AMG

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

1 Find $\int \cos 2x\, dx$.

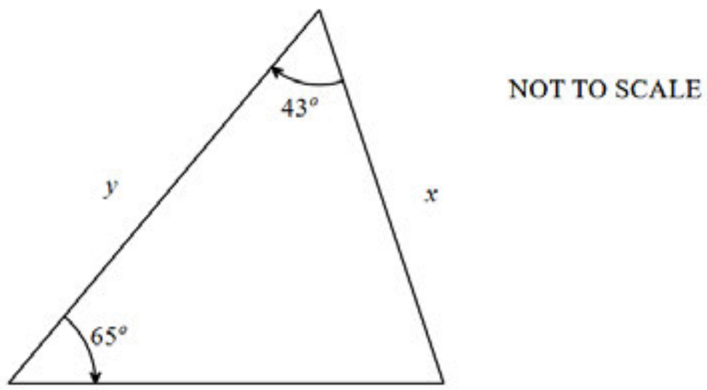
A. $\frac{-1}{2}\sin 2x + c$

B. $\frac{1}{2}\sin 2x + c$

C. $2\sin 2x + c$

D. $\sin 4x + c$

2 From the diagram, what is the value of y?



A. $y = \frac{x\sin 65^\circ}{\sin 72^\circ}$

B. $y = \frac{x\sin 43^\circ}{\sin 65^\circ}$

C. $y = \frac{x\sin 72^\circ}{\sin 65^\circ}$

D. $y = \frac{x\sin 72^\circ}{\sin 43^\circ}$

3 The set of ordered pairs is a relation.

$$\{(-2, 3), (-4,5), (3, 2), (-2, 1), (7, 2)\}.$$

Which of the following types is this relation?

A. one to one

B. one to many

C. many to many

D. many to one

4 A circle has the equation $(x+2)^2+(y+3)^2=d$.

What is the value of d such that the x-axis is a tangent to the circle?

A. 2

B. 3

C. 4

D. 9

5 If $y=3^{4x-5}$, then which of the following is $\dfrac{dy}{dx}$?

A. $4\left(3^{4x-5}\right)\ln 3$

B. $3\left(3^{4x-5}\right)\ln 3$

C. $4\left(3^{4x-5}\right)\ln 4$

D. $4\left(3^{4x-4}\right)\ln 3$

6 The cost per student on a school bus excursion varies inversely with the number of students travelling. If 72 students go on the excursion the cost per student is $126.

What is the cost per student if 168 students go on the excursion?

A. $294

B. $65

C. $60

D. $54

7 Which of the following is the range of the function $g(x)=1+2\sqrt{4-x^2}$?

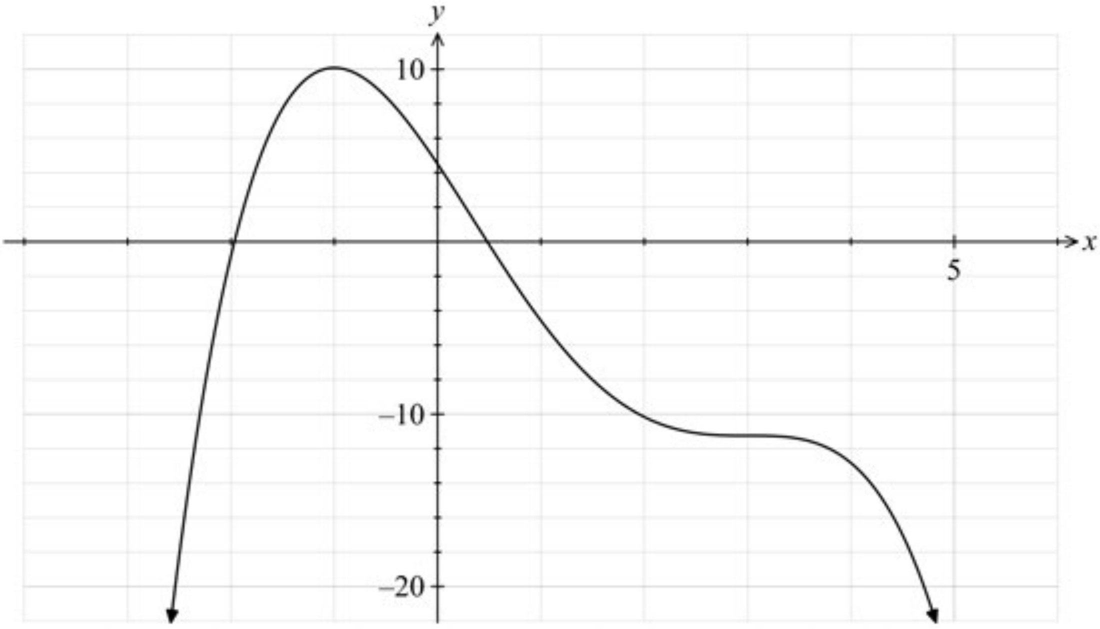
A. [1, 3]

B. [0, 2]

C. [1, 5]

D. [0, 5]

8 The diagram shows the graph of $y = h(x)$.



Which of the following could be the equation of $h'(x)$?

A. $h'(x) = (x+1)(x-3)^2$

B. $h'(x) = -(x+1)(x-3)^2$

C. $h'(x) = (x+1)(x-2)(x-4)$

D. $h'(x) = (x+1)(x-2)(4-x)$

9 The function $g(x) = 4\cos 2x$ is to be dilated horizontally by a factor of $\frac{1}{4}$ and then translated horizontally $\frac{\pi}{6}$ units to the right.

What is the equation of the transformed function $h(x)$?

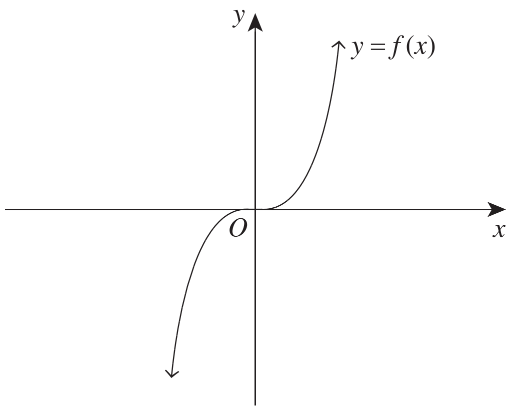
A. $h(x) = 4\cos\frac{1}{2}\left(x - \frac{\pi}{6}\right)$

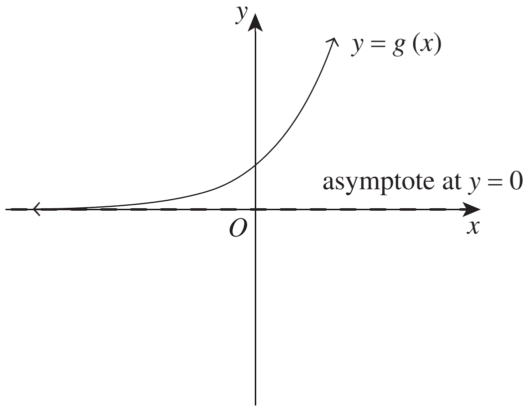
B. $h(x) = 4\cos\left(\frac{1}{2}x - \frac{\pi}{6}\right)$

C. $h(x) = 4\cos 8\left(x - \frac{\pi}{6}\right)$

D. $h(x) = 4\cos\left(8x - \frac{\pi}{6}\right)$

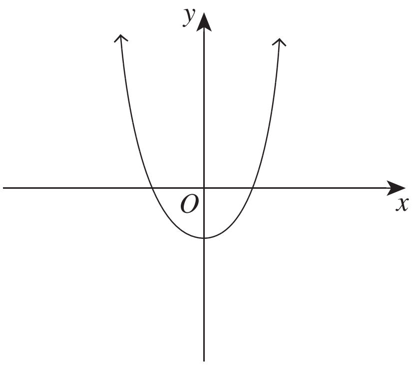
10 The graphs of $f(x)$ and $g(x)$ are shown.





Which of the following best represents the graph of $y = f\left(g(-x)\right)$?

A.



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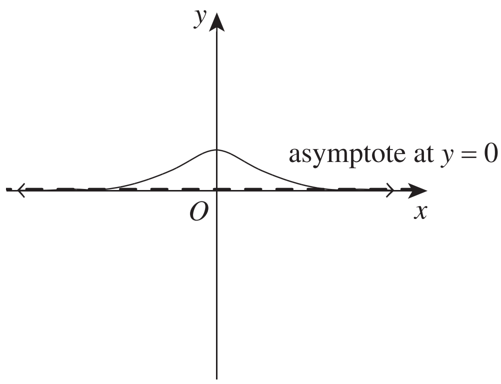
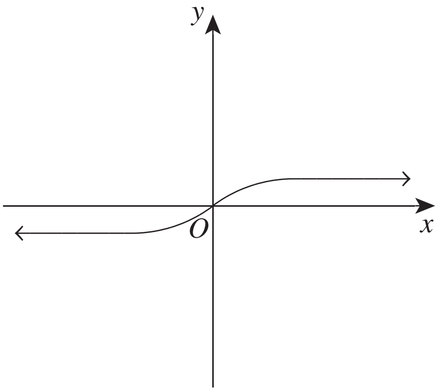
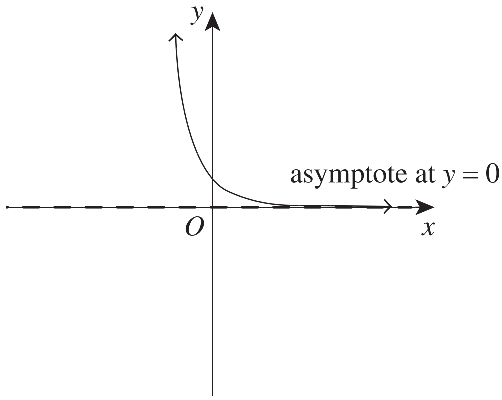


C.



D.



NESA Number									

YEAR 12 Mathematics Advanced

Cohort Task #4 (THSC)

Part A

Section II

Part A **15 marks**
Attempt Questions 11–16

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Make b the subject of the formula $a(b+5) = 2a + b$. **2**

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Question 12 (2 marks)

Differentiate $\dfrac{2x}{3x^2+1}$. **2**

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Question 13 (4 marks)

(a) Find $\int \sqrt[3]{4x+7}\, dx$. **2**

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(b) Evaluate $\int_0^{\frac{\pi}{6}} \frac{1}{\cos^2 x}\, dx$. **2**

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Question 14 (3 marks)
The following table gives the monthly repayment for each $1 000 borrowed.

Term of Loan (years)	**6.00 %**	**6.25 %**	**6.50 %**	**6.75 %**	**7.00 %**	**7.25 %**	**7.50 %**
5	$19.33	$19.45	$19.57	$19.68	$19.80	$19.92	$20.04
10	$11.10	$11.23	$11.35	$11.48	$11.61	$11.74	$11.87
15	$8.44	$8.57	$8.71	$8.85	$8.99	$9.13	$9.27
20	$7.16	$7.31	$7.46	$7.60	$7.75	$7.90	$8.06
25	$6.44	$6.60	$6.75	$6.91	$7.07	$7.23	$7.39

(a) Derek decides to travel during his gap year. He borrows $20 000 at 6.75% p.a. from the bank. **1**
If he will repay the loan over 15 years, what is Derek's monthly repayment?

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(b) How much interest does Derek pay on the loan? **2**

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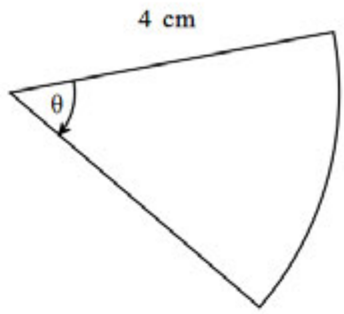
Question 15 (2 marks)

The radius and the area of the sector below are 4 cm and 3π cm^2 respectively. **2**



Calculate the exact perimeter of the sector in terms of π.

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Question 16 (2 marks)

Given that $\log_m p = 1.75$ and $\log_m q = 2.25$, find an expression for $\sqrt[5]{pq^2}$ **2**
leaving your answer in terms of m.

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End of Part A

Use this space to re-write any questions for Part A.

NESA Number											

YEAR 12 Mathematics Advanced

Cohort Task #4 (THSC)

Part B

Section II

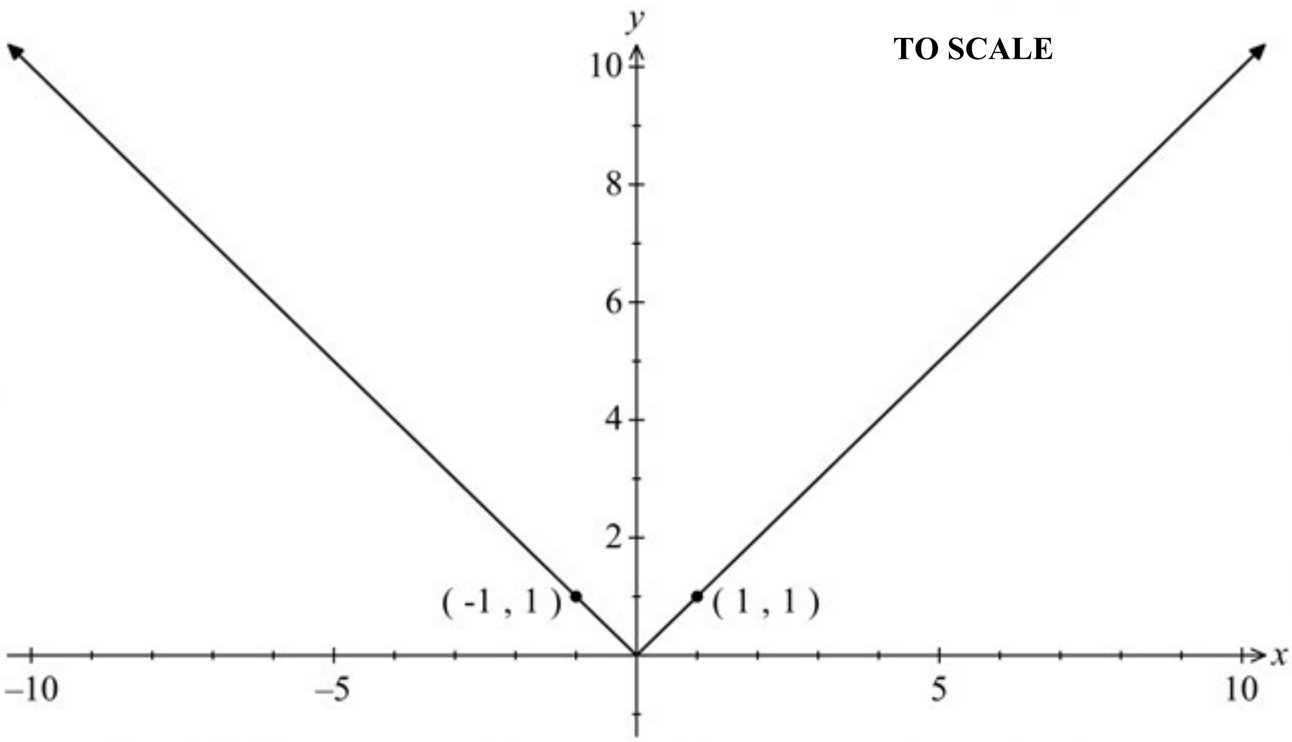
Part B **15 marks**
Attempt Questions 17–19

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 17 (2 marks)

The graph of the function, $f(x)$, is shown below. **2**

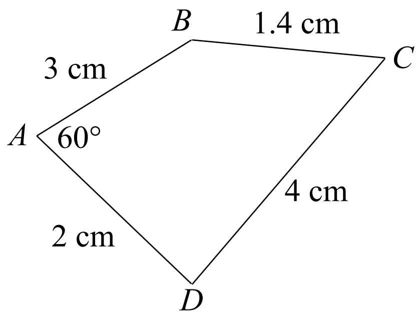


On the diagram above sketch $y = 2(f(x) + 2)$.

Question 18 (5 marks)

The shape of a new logo is shown below. **5**

$AB = 3$ cm, $BC = 1.4$ cm, $CD = 4$ cm, $AD = 2$ cm, and $\angle BAD = 60°$.



Find the area of the logo.
Leave your answer correct to one decimal place.

Question 19 (8 marks)

The gradient function of a function, $f(x)$, is given by $f'(x) = (3x-4)(x-4)$.

The curve $y = f(x)$ passes through (1 ,9).

(a) Find the equation of the curve $y = f(x)$. **2**

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(b) Find any stationary points and determine their nature. **2**

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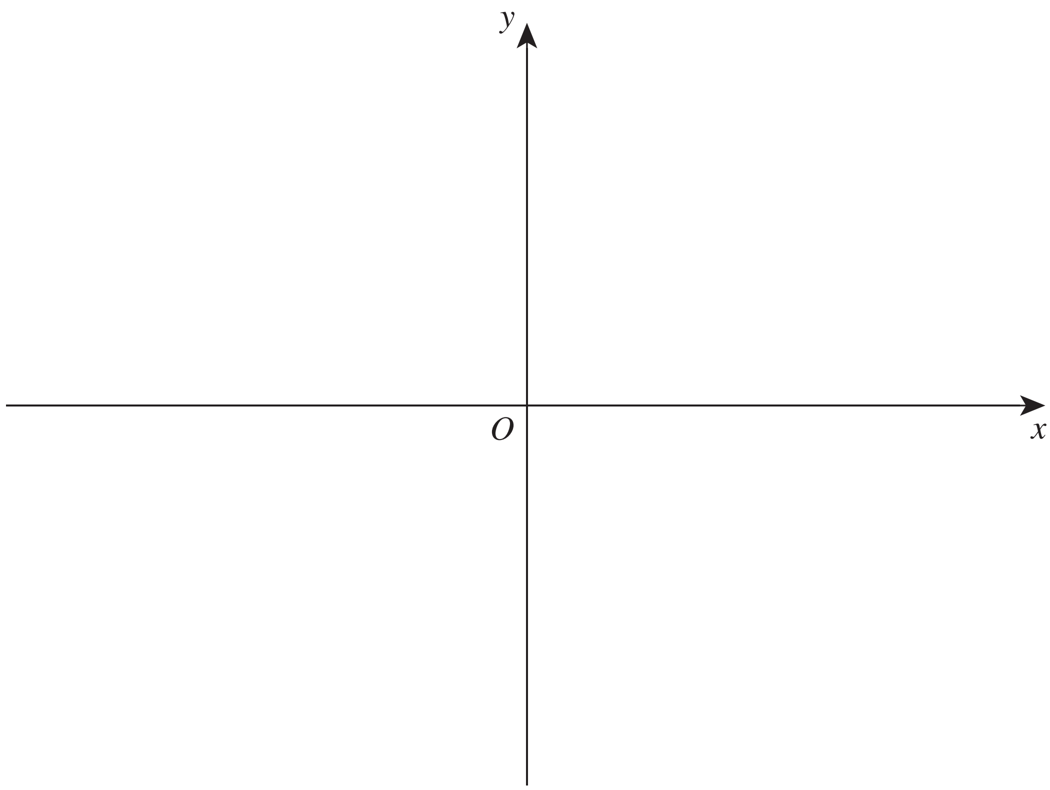
Question 18 continues on page 16

Question 19 (continued)

(c) Find any points of inflection. **2**

(d) Sketch the curve $y = f(x)$ clearly labelling turning points and intercepts. **2**



End of Part B

Use this space to re-write any questions for Part B.

NESA Number										

YEAR 12 Mathematics Advanced

Cohort Task #4 (THSC)

Part C

Section II

Part C **15 marks**
Attempt Questions 20–22

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 20 (7 marks)

Kate inherits \$100 000 and invests it in an account earning interest at a rate of 0.6% per month. Each month, immediately after the interest has been paid, Kate withdraws \$1 200.

The amount in the account immediately after the nth withdrawal can be determined using the recurrence relation

$$A_n = A_{n-1}(1.006) - 1200$$

where $A_0 = 100\,000$ and $n = 1, 2, 3, \ldots$.

(a) Apply the above recurrence relation to find the amount of money in the account immediately after the third withdrawal. **2**

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(b) Calculate the amount of interest earned in the first three months. **2**

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Question 21 continues on page 21

Question 20 (continued)

(c) Calculate the amount of money in the account immediately after the 75th withdrawal. **3**

Show all working.

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End of Question 20

Question 21 (5 marks)

The populations of two towns G and H are given respectively

$$P_G = 40\,000\, e^{k_1 t} \text{ for town } G \text{ and } P_H = 28\,000\, e^{k_2 t} \text{ for town } H,$$

where k_1 and k_2 are constants, t is the time in years which has elapsed since January 1st, 2010.

It is known that at the beginning of the year 2010, the populations were 40 000 for town G and 28 000 for town H.

At the beginning of 2013 the population of town G grew to 44 000 and the population of town H grew to 37 000.

(a) Find the value k_1 and k_2 correct to 4 decimal places. **2**

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(b) Find during which year the population of town H first exceeds the population of town G. **3**

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Question 22 (3 marks)

The share price ($\$P$) of a renewable energy company is increasing such that **3**

$$\frac{dP}{dt} = \frac{3}{t+1},$$

where t is the time in months.

If the initial share price is \$2.00, find the price after 6 months.
Write your answer to the nearest cent.

End of Part C

Use this space to re-write any questions for Part C.

NESA Number										

YEAR 12 Mathematics Advanced

Cohort Task #4 (THSC)

Part D

Section II

Part D 16 marks
Attempt Questions 23–26

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 23 (3 marks)

The seats in a theatre are numbered in numerical order from the first row to the last row.
The first row has 14 seats. Each successive row has 4 more seats than the previous one.

(a) How many seats are there in the fourth row? **1**

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(b) The theatre has a maximum capacity of 1320 seats, what is the greatest number of rows of seats in the theatre? **2**

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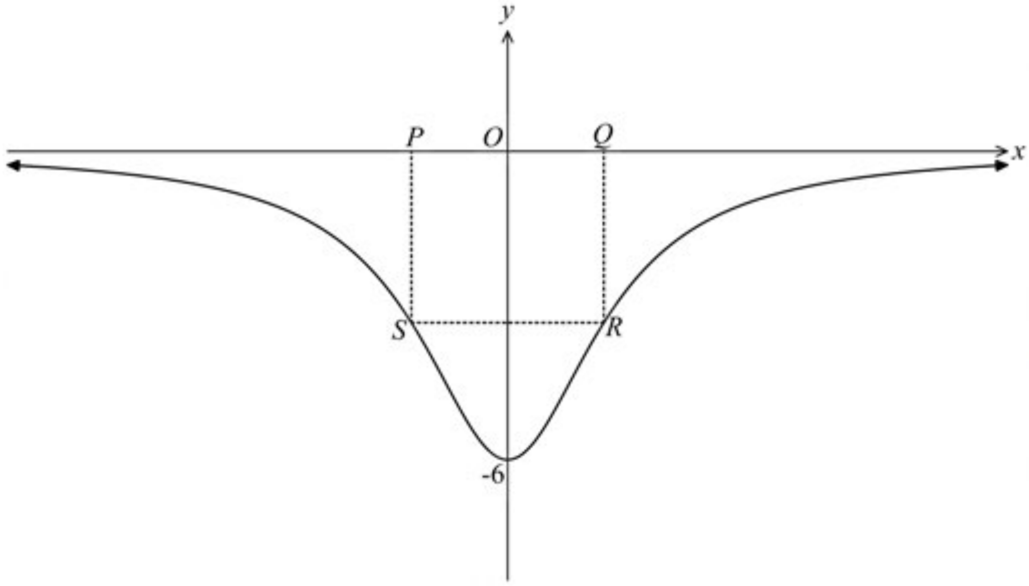
Question 24 (5 marks)



In the diagram above, the curve $y = -\dfrac{30}{x^2+5}$ has a minimum turning point at (0, –6).

A rectangle *PQRS* is inscribed within the curve as shown with its axis of symmetry $x = 0$.

Q has coordinates $(\alpha, 0)$.

(a) Show that the area *A*, of the rectangle *PQRS* is $A = \dfrac{60\alpha}{\alpha^2+5}$. **2**

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Question 24 continues on page 27

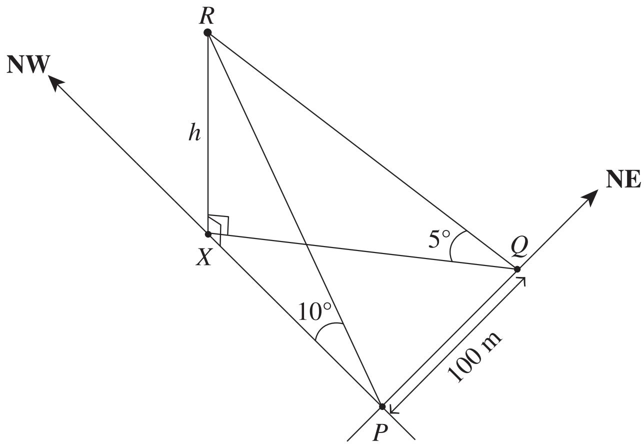
Question 24 (continued)

(b) Show that the maximum area of the rectangle *PQRS* is $6\sqrt{5}$ square units. **3**

End of Question 24

Question 25 (4 marks)

A hiker starts from point P and walks 100 metres along a straight path to the north-east, arriving at point Q. A peak is located at point R, which is h metres directly above point X.
From point P the angle of elevation to peak R is 10°. At point Q, the angle of elevation to the peak is 5°.



NOT TO SCALE

(a) Show that $XP = h \cot 10°$, and write down a similar expression for XQ. **1**

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(b) Hence, find the value of h. Give your answer correct to the nearest metre. **3**

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Question 26 (4 marks)

A particle moves from rest at a point A in a straight line, so that t seconds after leaving A its displacement, x metres, is given by $x = 16t^2 - 2t^4$.

Find

(a) the time(s) for the particle to come to rest again. **2**

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(b) the distance travelled in the first three seconds. **2**

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End of Part D

Use this space to re-write any questions for Part D.

NESA Number											

YEAR 12 Mathematics Advanced

Cohort Task #4 (THSC)

Part E

Section II

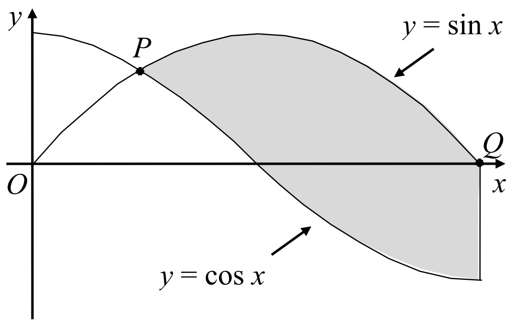
Part E 15 marks
Attempt Questions 27–29

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 27 (5 marks)

The graph below shows the curves $y = \sin x$ and $y = \cos x$.



(a) Find the x-coordinates of the points P and Q. **2**

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(b) Find the area of the shaded region. **3**

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Question 28 (3 marks)

A bag contains four red balls, two black balls, and one white ball.

Wyndham selects one ball from the bag and keeps it hidden.

He then selects a second ball, also keeping it hidden.

(a) Find the probability that both the selected balls are red. **1**

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(b) Wyndham drops one of the selected balls and we can see that it is red. **2**
What is the probability that the ball that is still hidden is also red?

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Question 29 (7 marks)

On a given day the depth of water in a river is modelled by the function

$$h(t) = 5 + 3\sin\left(\frac{\pi t}{4}\right),$$

where h is the depth of the water, in metres, and t is the time, in hours, after 12 midnight.

(a) What is the depth of the water at 12 midnight? **1**

(b) Sketch the graph of $h(t) = 5 + 3\sin\left(\frac{\pi t}{4}\right)$ in the domain [0, 24]. **3**

(c) A family decides to go on a picnic by the river from 12 midday to 2 pm. **3**
It is only safe to swim in the river if the depth of water is less than 4 metres.
When is the earliest time the family should swim after 12 midday?
Give your answer correct to the nearest minute.

End of Part E

Use this space to re-write any questions for Part E.

NESA Number											

YEAR 12 Mathematics Advanced

Cohort Task #4 (THSC)

Part F

Section II

Part F 14 marks
Attempt Questions 30–32

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 30 (2 marks)

Evaluate $\lim\limits_{x \to 4} \dfrac{x^2 - 2x - 8}{\sqrt{x} - 2}$. **2**

Show all working.

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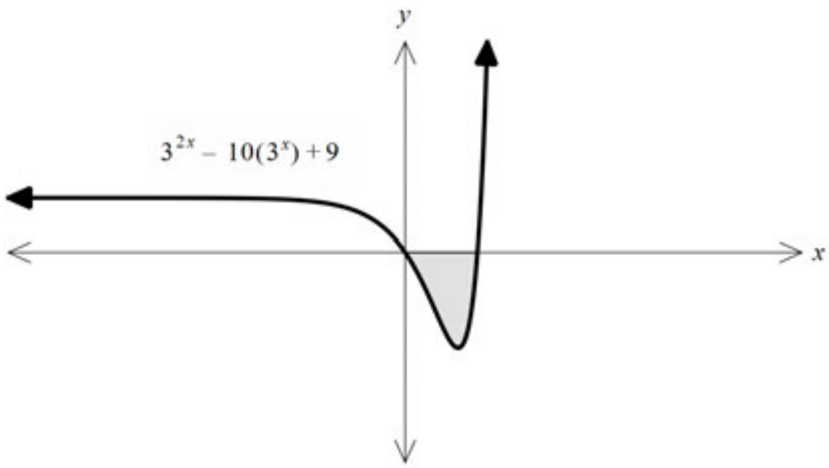
Question 31 (5 marks)

The graph of $f(x) = 3^{2x} - 10\left(3^x\right) + 9$ is shown below. **5**

Find the exact value of the area shaded.



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Question 32 (7 marks)

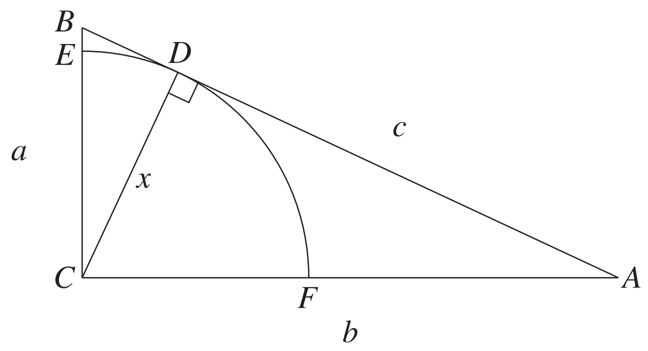
In triangle ABC, BC is perpendicular to AC.

Side BC has length a, side AC has length b and side AB has length c.

A quadrant of a circle of radius x, centred at C, is constructed.

The arc meets side BC at E. It touches the side AB at D, and meets side AC at F.

The interval CD is perpendicular to AB.



(a) Explain why $x = \dfrac{ab}{c}$ **2**

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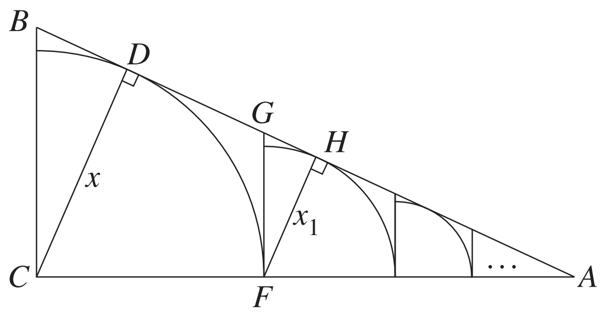
Question 32 continues on page 40

Question 32 (continued)

From F, a line perpendicular to AC is drawn to meet AB at G, forming the right-angled triangle GFA.

A new quadrant is constructed in triangle GFA touching side AB at H.

The process is then repeated indefinitely.



(b) Show that the limiting sum of the areas of all the quadrants is $\dfrac{\pi a b^2}{4(2c - a)}$. **4**

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Question 32 continues on page 41

Question 32 (continued)

(c) Hence, or otherwise, show that $\frac{\pi}{2} < \frac{2c - a}{b}$. **1**

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End of paper

Use this space to re-write any questions for Part F.

SYDNEY
BOYS
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SCHOOL

2023

YEAR 12

TASK 4 – THSC

Mathematics Advanced

Sample Solutions

NOTE: Some of you may be disappointed with your mark.
This process of checking your mark is NOT the opportunity to improve your marks.
Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done. Writing something down does not justify that your working can be linked to a mark.

Students who used pencil, an erasable pen and/or whiteout, may NOT be able to appeal.

MC Answers

1	B	**6**	D
2	C	**7**	C
3	C	**8**	B
4	D	**9**	C
5	A	**10**	D

Section I – Multiple Choice

1 Find $\int \cos 2x \, dx$.

A. $\frac{-1}{2}\sin 2x + c$

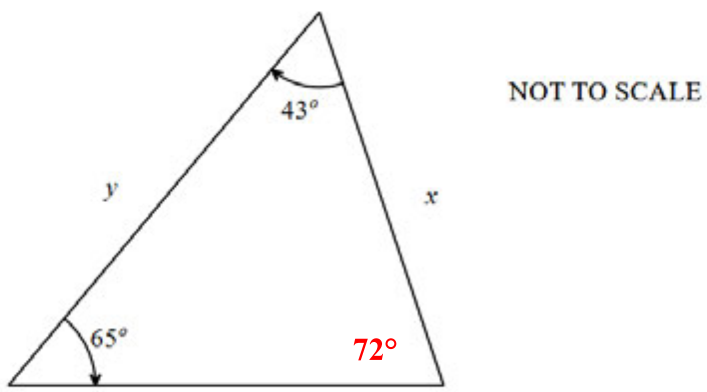
B. $\frac{1}{2}\sin 2x + c$

C. $2\sin 2x + c$

D. $\sin 4x + c$

$$\int f'(x)\cos(f(x))\,dx = \sin(f(x)) + C$$

2 From the diagram, what is the value of y?



A. $y = \frac{x\sin 65^\circ}{\sin 72^\circ}$

B. $y = \frac{x\sin 43^\circ}{\sin 65^\circ}$

C. $y = \frac{x\sin 72^\circ}{\sin 65^\circ}$

D. $y = \frac{x\sin 72^\circ}{\sin 43^\circ}$

Sine rule: $\frac{y}{\sin 72^\circ} = \frac{x}{\sin 65^\circ}$

3 The set of ordered pairs is a relation.

$$\{(-2, 3), (-4,5), (3, 2), (-2, 1), (7, 2)\}.$$

Which of the following types is this relation?

A. one to one

B. one to many

C. many to many

D. many to one

Shared x-coordinates & y-coordinates.

4 A circle has the equation $(x+2)^2+(y+3)^2=d$.

What is the value of d such that the x-axis is a tangent to the circle?

A. 2

B. 3

C. 4

D. 9

The radius is equal to the distance from the x-axis to the centre i.e. $r=3$

$d=r^2=9$

5 If $y=3^{4x-5}$, then which of the following is $\frac{dy}{dx}$?

A. $4\left(3^{4x-5}\right)\ln 3$

B. $3\left(3^{4x-5}\right)\ln 3$

C. $4\left(3^{4x-5}\right)\ln 4$

D. $4\left(3^{4x-4}\right)\ln 3$

$$y=a^{f(x)} \Rightarrow \frac{dy}{dx}=(f'(x)\ln a)\ a^{f(x)}$$

6 The cost per student on a school bus excursion varies inversely with the number of students travelling. If 72 students go on the excursion the cost per student is \$126.

What is the cost per student if 168 students go on the excursion?

A. \$294

B. \$65

C. \$60

D. \$54

With inverse variation $xy=k$.

$\therefore k=72\times 126=9072$

$\therefore 168\times y=9072$

7 Which of the following is the range of the function $g(x)=1+2\sqrt{4-x^2}$?

A. [1, 3]

B. [0, 2]

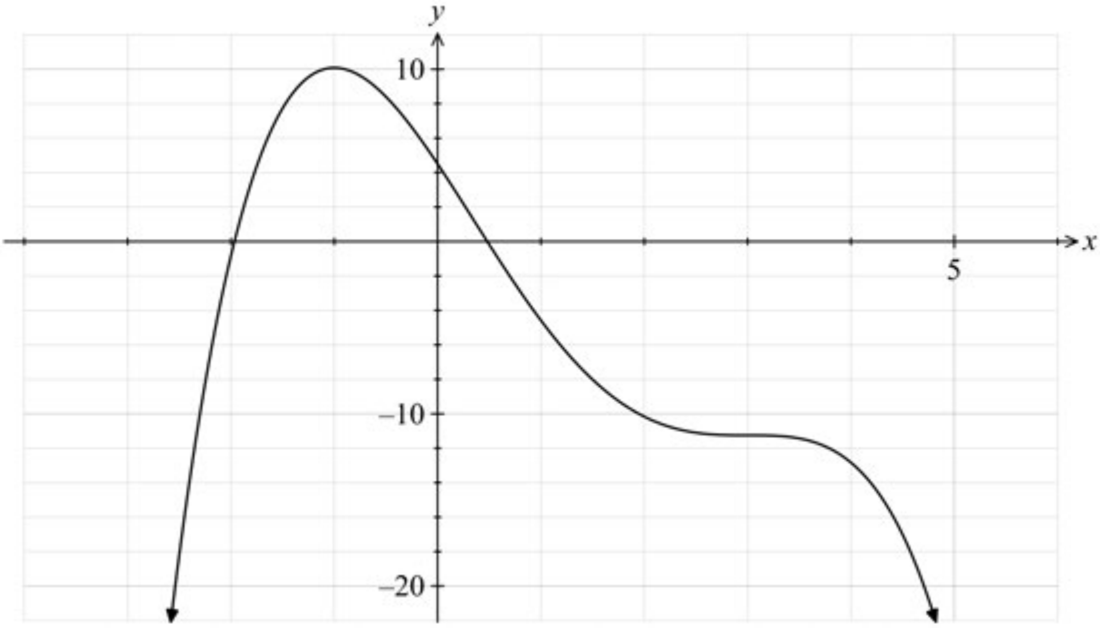
C. [1, 5]

D. [0, 5]

$$0\le\sqrt{4-x^2}\le 2 \Rightarrow 0\le 2\sqrt{4-x^2}\le 4$$

$$\therefore 1+2\sqrt{4-x^2}\le 5$$

8 The diagram shows the graph of $y = h(x)$.



Which of the following could be the equation of $h'(x)$?

A. $h'(x) = (x+1)(x-3)^2$

B. $h'(x) = -(x+1)(x-3)^2$

C. $h'(x) = (x+1)(x-2)(x-4)$

D. $h'(x) = (x+1)(x-2)(4-x)$

There are only two stationary points.
$\therefore$ A or B
The graph is 'upside down'
$\therefore$ B

9 The function $g(x) = 4\cos 2x$ is to be dilated horizontally by a factor of $\frac{1}{4}$ and then translated horizontally $\frac{\pi}{6}$ units to the right.

What is the equation of the transformed function $h(x)$?

A. $h(x) = 4\cos\frac{1}{2}\left(x - \frac{\pi}{6}\right)$

B. $h(x) = 4\cos\left(\frac{1}{2}x - \frac{\pi}{6}\right)$

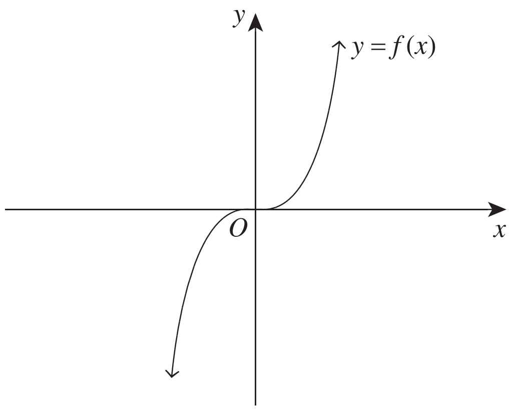
C. $h(x) = 4\cos 8\left(x - \frac{\pi}{6}\right)$

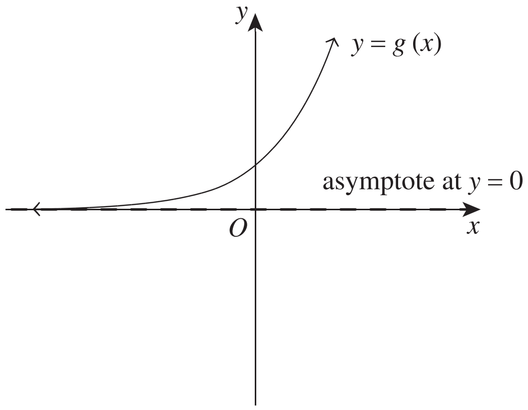
D. $h(x) = 4\cos\left(8x - \frac{\pi}{6}\right)$

Let $f(x) = g(4x) = 4\cos 8x$

$\therefore h(x) = f\left(x - \frac{\pi}{6}\right) = 4\cos 8\left(x - \frac{\pi}{6}\right)$

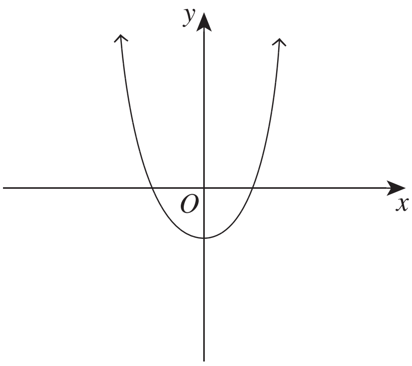
10 The graphs of $f(x)$ and $g(x)$ are shown.



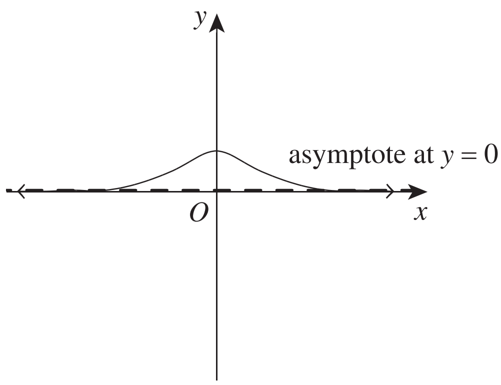


Which of the following best represents the graph of $y = f\left(g(-x)\right)$?

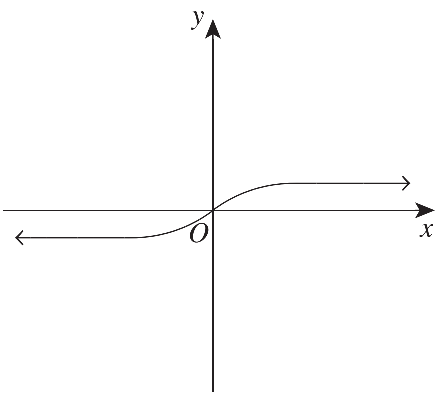
A.



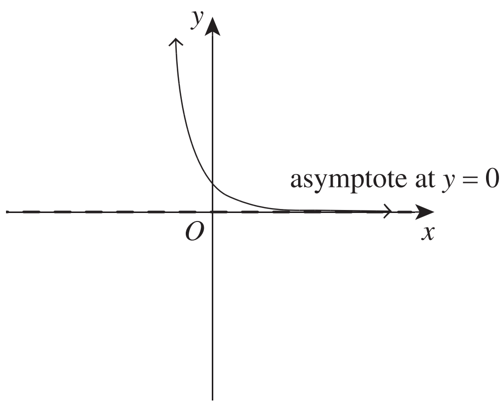
B.



C.



D.



$g(-x)$ is a reflection in the y-axis.

So $g(-x) \rightarrow 0$ as $x \rightarrow \infty$. As $x \rightarrow 0, f(x) \rightarrow 0$

So B or D.

D is the only graph that is neither ODD nor EVEN

NESA Number	M	A	R	K	I	N	G		

SCHEME

15

YEAR 12 Mathematics Advanced

Cohort Task #4 (THSC)

Part A

Section II

Part A **15 marks**
Attempt Questions 11–16

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Make b the subject of the formula $a(b+5)=2a+b$. **2**

$ab+5a=2a+b$ — $\frac{1}{2}$

$ab-b=-3a$ OR $b-ab=3a$ — $\frac{1}{2}$

$b(a-1)=-3a$ OR $b(1-a)=3a$ $\frac{1}{2}$

$b=\frac{-3a}{a-1}$ OR $b=\frac{3a}{1-a}$ $\frac{1}{2}$

$b=\frac{-3}{1-\frac{1}{a}}$ also accepted

Question 12 (2 marks)

Differentiate $\frac{2x}{3x^2+1}=y$ **2**

$u=2x$, $v=3x^2+1$

$u'=2$

$v'=6x$

$y'=\frac{dy}{dx}=f'(x)=\frac{vu'-uv'}{v^2}$ $\frac{1}{2}$

$=\frac{2(3x^2+1)-2x(6x)}{(3x^2+1)^2}$ $\frac{1}{2}$

$=\frac{6x^2+2-12x^2}{(3x^2+1)^2}$

$=\frac{-6x^2+2}{(3x^2+1)^2}$ OR $-\frac{6x^2-2}{(3x^2+1)^2}$ 1

OR $\frac{2-6x^2}{(\quad)^2}$ OR $\frac{2(1-3x^2)}{(\quad)^2}$ OR $\frac{-2(3x^2-1)}{(\quad)^2}$

$y'=\frac{2}{(3x^2+1)}-\frac{12x^2}{(3x^2+1)^2}$ also accepted

Question 13 (4 marks)

(a) Find $\int \sqrt[3]{4x+7}\,dx$. **2**

$= \int (4x+7)^{\frac{1}{3}}\,dx$ $\frac{1}{2}$

$= \frac{(4x+7)^{\frac{4}{3}}}{\frac{4}{3}\times 4} + C$ 1

$= \frac{3(4x+7)^{\frac{4}{3}}}{16} + C$ or $\frac{3}{16}\sqrt[3]{(4x+7)^4} + C$ $\frac{1}{2}$

While the +C is compulsory & loses $-\frac{1}{2}$m, having it there just for the sake of it because it's integration doesn't get you anything.

(b) Evaluate $\int_0^{\frac{\pi}{6}} \left(\frac{1}{\cos^2 x}\right) dx$. $= \int_0^{\frac{\pi}{6}} \sec^2 x\,dx$ $\frac{1}{2}$ **2**

$= \left[\tan x\right]_0^{\frac{\pi}{6}=30°}$ $\frac{1}{2}$

$= \tan 30° - \tan 0°$ $\frac{1}{2}$

$= \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ $\frac{1}{2}$

OR (let's assume you were unfortunate to not remember $\boxed{\frac{1}{\cos x} = \sec x}$

then $\int_0^{\frac{\pi}{6}} \frac{dx}{\cos^2 x} = \int_0^{\frac{\pi}{6}} \frac{\cos^2 x + \sin^2 x}{\cos^2 x}\,dx$

$= \int_0^{\frac{\pi}{6}} (1 + \tan^2 x)\,dx$

$= \int_0^{\frac{\pi}{6}} \sec^2 x\,dx$

$=$ etc.....

- other answers are clearly wrong but can gain partial marks based on working.

Question 14 (3 marks)

The following table gives the monthly repayment for each \$1 000 borrowed.

Term of Loan (years)	**6.00 %**	**6.25 %**	**6.50 %**	**6.75 %**	**7.00 %**	**7.25 %**	**7.50 %**
5	\$19.33	\$19.45	\$19.57	\$19.68	\$19.80	\$19.92	\$20.04
10	\$11.10	\$11.23	\$11.35	\$11.48	\$11.61	\$11.74	\$11.87
15	\$8.44	\$8.57	\$8.71	\$8.85	\$8.99	\$9.13	\$9.27
20	\$7.16	\$7.31	\$7.46	\$7.60	\$7.75	\$7.90	\$8.06
25	\$6.44	\$6.60	\$6.75	\$6.91	\$7.07	\$7.23	\$7.39

(a) Derek decides to travel during his gap year. He borrows \$20 000 at 6.75% p.a. from the bank. **1**

If he will repay the loan over 15 years, what is Derek's monthly repayment?

$\div 1000$

$8.85 \times 20 = \$177$ $\frac{1}{2} + \frac{1}{2}$

ECF

(b) How much interest does Derek pay on the loan? **2**

total paid $= 177 \times 12 \times 15$

$= 31860$ (1)

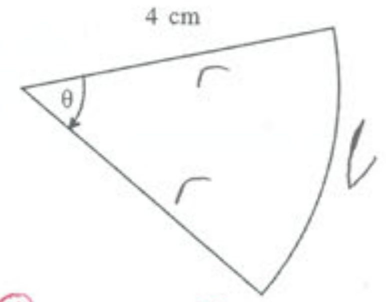
Interest = total paid − amt. borrowed

$= 31860 - 20000$

$I = \$11860$ (1)

Question 15 (2 marks)

The radius and the area of the sector below are 4 cm and 3π cm^2 respectively. **2**



Calculate the <u>exact perimeter</u> of the sector in terms of π. 1 1

$P = 2r + l$ → what is θ? not given.

$= 2\times4 + r\theta$

$= 8 + 4\times\frac{3\pi}{8}$ $\qquad A = 3\pi$

$= (8 + \frac{3\pi}{2})$ cm $\qquad A = \frac{1}{2}\theta r^2$

$\frac{\theta}{2}\times4^2 = 3\pi$

$8\theta = 3\pi$

$\theta = \frac{3\pi}{8}$ $(=67.5°C)$

$\frac{1}{2}$

Question 16 (2 marks)

Given that $\log_m p = 1.75$ and $\log_m q = 2.25$, find an expression for $\sqrt[5]{pq^2}$ leaving your answer in terms of m. **2**

$p = m^{1.75} = m^{\frac{7}{4}}$ $\qquad q = m^{2.25} = m^{\frac{9}{4}}$ $\quad \frac{1}{2}$

method 1: $\sqrt[5]{pq^2} = (pq^2)^{\frac{1}{5}}$ — OR 2: $\left(m^{\frac{7}{4}}\times\left(m^{\frac{9}{4}}\right)^2\right)^{\frac{1}{5}}$

$= p^{\frac{1}{5}}q^{\frac{2}{5}}$ $\qquad = \left(m^{\frac{7}{4}}\times m^{\frac{18}{4}}\right)^{\frac{1}{5}}$

$= \left(m^{\frac{7}{4}}\right)^{\frac{1}{5}}\left(m^{\frac{9}{4}}\right)^{\frac{2}{5}}$ $\qquad = \left(m^{\frac{25}{4}\times\frac{1}{5}}\right)$ $\quad 1$

$= m^{\left(\frac{7}{20}+\frac{18}{20}\right)}$ $\qquad = m^{\frac{5}{4}}$

$= m^{\frac{25}{20}}$ $\qquad = m^{1.25}$

$= m^{\frac{5}{4}}$ OR $\sqrt[4]{m^5}$ $\quad \frac{1}{2}$

PTO for more brilliant & creative solutions

End of Part A

Use this space to re-write any questions for Part A.

(16) method (3): $\left(m^{1.75} \times \left(m^{2.25}\right)^2\right)^{\frac{1}{5}} = \left(m^{1.75+4.5}\right)^{\frac{1}{5}}$

$= m^{6.25 \div 5}$

$= m^{1.25}$

(4) $p^{\frac{1}{5}} q^{\frac{2}{5}} = m^{\log_m\left(p^{\frac{1}{5}} \times q^{\frac{2}{5}}\right)}$

$= m^{\left(\log_m p^{\frac{1}{5}} + \log_m q^{\frac{2}{5}}\right)}$

$= m^{\left(\frac{1}{5}\log_m p + \frac{2}{5}\log_m q\right)}$

$= m^{\left(\frac{1}{5} \times 1.75 + \frac{2}{5} \times 2.25\right)}$

$= m^{(0.35+0.9)}$

$=$ ......

(5) $\log_m(pq^2) = \log_m p + 2\log_m q = 6.25$

$\therefore pq^2 = m^{6.25}$

$\sqrt[5]{pq^2} = m^{\left(\frac{6.25}{5}\right)}$

$=$ ......

Section B Feedback

Q17. This question was generally done 'mostly well' with many students dropping $\frac{1}{2}$ a mark for misinterpreting the translation as 'horizontal' instead of 'vertical'.
Many students also forgot to label a point other than the intercept, which was also $\frac{1}{2}$ mark short of 2/2.

Q18. This question was done very well by the majority of students, with most errors being either calculation errors or forgetting to use degrees (instead of radians).
A fair few students assumed that the diagram was to scale and assumed info that wasn't given on the diagram (eg. $\angle DBC = 90$).
The key to solving this problem was using the cosine rule (for BD & $\angle C$) and the area formula ($A = \frac{1}{2} ab \sin C$).

Q19. a) Generally done very well. Some students forgot to show that $c=0$, thus having an incomplete solution.
b) Generally done very well, most students used the derivatives to identify x-vals and nature as intended.
c) Mostly done well, but many students ~~did~~ did not check whether the point at $x=\frac{8}{3}$ was actually a point of inflexion.

d) Mostly done well with errors mostly coming from using a horizontal inflexion instead of an ~~vert~~ oblique one. Egregious lapses in concavity were also observed.

Section II

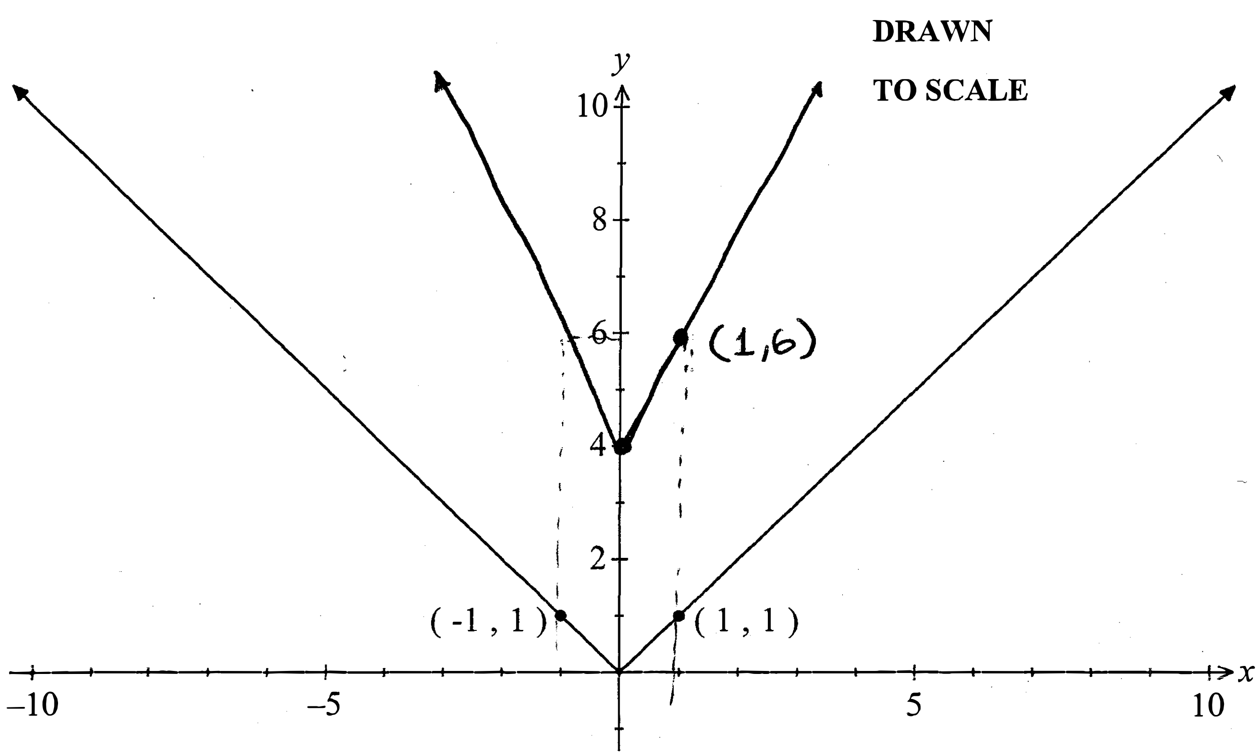
Part B **15 marks**
Attempt Questions 17–19

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 17 (2 marks)

The graph of the function, $f(x)$, is shown below. **2**



On the diagram above sketch $y = 2(f(x) + 2)$.

Method 1: Observe that the transformation is linear, so we can evaluate the transformation of points $(0,0)$ and $(1,1)$.

Letting $g(x) = 2(f(x)+2)$

$g(0) = 4$, $g(1) = 6$

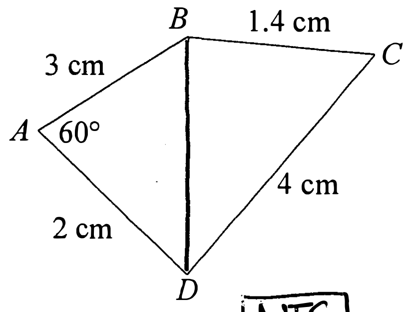
$\therefore$ graph has vertex $(0,4)$ and passes through $(1,6)$.

Method 2: $y = 2(f(x)+2)$ is a vertical translation (of 2 units) followed by a vertical dilation (by a factor of 2).

Question 18 (5 marks)

The shape of a new logo is shown below. **5**

$AB = 3$ cm, $BC = 1.4$ cm, $CD = 4$ cm, $AD = 2$ cm, and $\angle BAD = 60°$.



$\boxed{\text{NTS}}$.

Find the area of the logo.
Leave your answer correct to one decimal place.

Construct BD,

using cosine rule: $BD = \sqrt{3^2 + 2^2 - 2(3)(2)\cos(60°)} = \sqrt{7}$

using cosine rule: $\angle C = \cos^{-1}\left(\frac{-\sqrt{7}^2 + 4^2 + 1.4^2}{2(4)(1.4)}\right) \approx 11.88°$

using $\text{Area}_{\Delta} = \frac{1}{2}ab\cdot\sin(C)$

$\text{Area}_{\Delta ABD} = \frac{1}{2}(3)(2)\sin(60°) = \frac{3\sqrt{3}}{2} \approx 2.598...$

$\text{Area}_{\Delta BCD} \approx \frac{1}{2}(4)(1.4)\sin(11.88°) \approx 0.576...$

$\therefore \text{Area}_{ABCD} = \frac{1}{2}(3)(2)\sin(60°) + \frac{1}{2}(4)(1.4)\sin(11.88°)$

$\approx 3.17449...$

≈ 3.2 (1 d.p)

Question 19 (8 marks)

The gradient function of a function, $f(x)$, is given by $f'(x) = (3x-4)(x-4)$.
The curve $y = f(x)$ passes through (1 ,9).

(a) Find the equation of the curve $y = f(x)$.

$f'(x) = (3x-4)(x-4)$

$= 3x^2 - 16x + 16$

using reverse power rule;

$f(x) = x^3 - 8x^2 + 16x + C.$

given that $f(1) = 9$,

we have: $(1)^3 - 8(1)^2 + 16(1) + C = 9$, $\therefore C = 0$

$\therefore f(x) = x^3 - 8x^2 + 16x.$

(b) Find any stationary points and determine their nature. **2**

stationary points are at: $f'(x) = (3x-4)(x-4) = 0$

$\therefore$ ~~st~~ we have stationary points at

$x = \frac{4}{3}$ & $x = 4$.

(sub into $y=f(x)$) $y = \frac{256}{27}$ & $y = 0$. (y-values required)

to determine nature, can use; table of values ~~for~~ or $f''(x)$.

eg. $f''(x) = 6x - 16$, $f''(\frac{4}{3}) = 8 - 16 < 0$, $\therefore$ max.

$f''(4) = 24 - 16 > 0$, $\therefore$ min.

hence; max point @ $(\frac{4}{3}, \frac{256}{27})$, min point @ $(4, 0)$

Question 18 continues on page 16

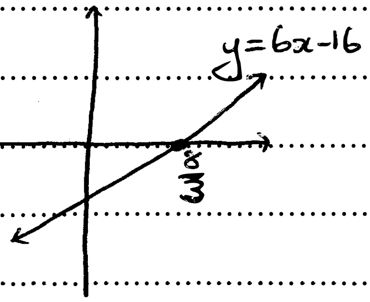
(c) Find any points of inflection. **2**

points of inflexion are at: $f''(x) = 6x - 16 = 0$.

$\therefore$ possible point of inflexion at $x = \frac{8}{3}$, $y = \frac{128}{27}$.

can also observe that $\frac{8}{3} = \frac{(\frac{4}{3}+4)}{2}$ which is turning point of $f'(x$

we check that it is indeed a point of inflexion using either a graph or sign table;



or

x	$\frac{7}{3}$	$\frac{8}{3}$	$\frac{9}{3}$
$f''(x)$	-2	0	2

Clearly $f''(x)$ changes sign, so we have inflexion at $x = \frac{8}{3}$

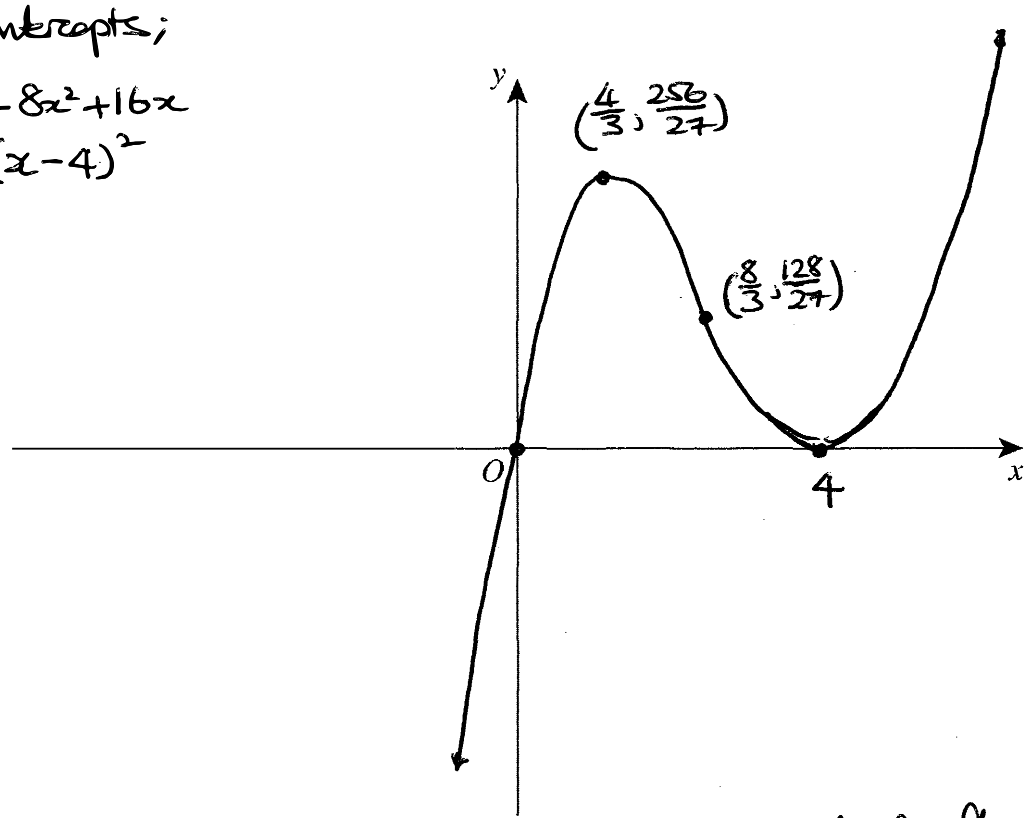
Change in concavity, so we confirm that there is a point of inflexion at $x = \frac{8}{3}$.

(d) Sketch the curve $y = f(x)$ clearly labelling turning points and intercepts. **2**

to find intercepts;

$f(x) = x^3 - 8x^2 + 16x$

$= x(x-4)^2$



note: point of inflexion not required.

End of Part B

Part C Solutions **15 marks**

Question 20 (7 marks)

Kate inherits \$100 000 and invests it in an account earning interest at a rate of 0.6% per month. Each month, immediately after the interest has been paid, Kate withdraws \$1 200.

The amount in the account immediately after the nth withdrawal can be determined using the recurrence relation

$$A_n = A_{n-1}(1.006) - 1200$$

where $A_0 = 100\,000$ and $n = 1, 2, 3, \ldots$.

(a) Apply the above recurrence relation to find the amount of money in the account immediately after the third withdrawal. **2**

$A_0 = 100000$

$A_1 = 100000 \times 1.006 - 1200 = 99400$

$A_2 = 99400 \times 1.006 - 1200 = 98796.4$

$A_3 = 98796.4 \times 1.006 - 1200 = 98189.1784 \approx 98189.18$

Well done.

To get full marks, sufficient working out is required.

Common error:

- *There are some boys only showed the expression for A_3 but did not calculate the value of A_3*
- *Some careless calculation mistakes.*

(b) Calculate the amount of interest earned in the first three months. **2**

Method 1:

$I = A_3 + 3 \times 1200 - A_0$

$= 98189.1784 + 3600 - 100000$

$= 1789.1784$

≈ 1789.18

Method 2:

$I_1 = 0.006 \times A_0 = 600$

$I_2 = 0.006 \times A_1 = 596.4$

$I_3 = 0.006 \times A_2 = 592.78$

$I = I_1 + I_2 + I_3 = 1789.1784$

Generally well done.

Common errors:

- $I = A_0 \times 1.006^3 - A_0 = 100000 \times 1.006^3 - 100000 \approx 1810.82$
- $I = A_0 - A_3 = 100000 - 98189.1784 = 1810.8216$
- $I = 0.006 \times (A_1 + A_2 + A_3) \approx 1778.31$
- *Some students did not read the question carefully and calculated the interest rate.*
- *Some careless calculation mistakes.*

Question 20 (continued)

(c) Calculate the amount of money in the account immediately after the 75^{th} withdrawal. **3**

Show all working.

$A_1 = A_0 \times 1.006 - 1200 = 1.006A_0 - 1200$

$A_2 = A_1 \times 1.006 - 1200 = 1.006(1.006A_0 - 1200) - 1200 = 1.006^2 A_0 - 1200(1 + 1.006)$

$A_3 = A_2 \times 1.006 - 1200$

$= 1.006[1.006^2 A_0 - 1200(1 + 1.006)] - 1200$

$= 1.006^3 A_0 - 1200(1 + 1.006 + 1.006^2)$

.

.

.

$\therefore A_n = 1.006^n A_0 - 1200(1 + 1.006 + 1.006^2 + \cdots + 1.006^{\boldsymbol{n-1}})$

$= 1.006^n \times 100000 - 1200 \times \frac{1.006^{\boldsymbol{n}} - 1}{1.006 - 1}$

$= 200000 - 100000 \times 1.006^n$

When $n = 75$

$A_n = 200000 - 100000 \times 1.006^{75} \approx 43379.52$

Generally well done, but some students did not attempt this question.

To get full marks, students need to

- *find the general formula A_n or A_{75} and*
- *demonstrate their abilities in calculating the sum of a geometric series S_n or S_{75}.*

Using trial-and-error, but without working out, is not sufficient to get full marks.

Common error:

- *Incorrectly identify the total term of the geometric series, see the highlighted parts above.*
- *Did not show ALL working*

End of Question 20

Question 21 (5 marks)

The populations of two towns G and H are given respectively

$$P_G = 40\,000\, e^{k_1 t} \text{ for town } G \text{ and } P_H = 28\,000\, e^{k_2 t} \text{ for town } H,$$

where k_1 and k_2 are constants, t is the time in years which has elapsed since January 1st, 2010.

It is known that at the beginning of the year 2010, the populations were 40 000 for town G and 28 000 for town H.

At the beginning of 2013 the population of town G grew to 44 000 and the population of town H grew to 37 000.

(a) Find the value k_1 and k_2 correct to 4 decimal places. **2**

When $t = 3$

$P_G = 40000e^{3k1} = 44000$

$e^{3k1} = \frac{11}{10}$

$lne^{3k1} = \ln\frac{11}{10}$

$3k_1 = \ln\frac{11}{10}$

$\therefore k_1 = \frac{\ln\frac{11}{10}}{3} \approx 0.0318\ (4d.p.)$

When $t = 3$

$P_H = 28000e^{3k2} = 37000$

$e^{3k2} = \frac{37}{28}$

$lne^{3k2} = \ln\frac{37}{28}$

$3k_2 = \ln\frac{37}{28}$

$\therefore k_2 = \frac{\ln\frac{37}{28}}{3} \approx 0.0929\ (4d.p.)$

Generally well done.

Common errors:

- *Incorrectly identified the value of t. Wrong answers include 2, 4, 13 etc.*
- *Unable to apply the knowledge of logarithm to find the value k_1 and k_2.*

Question 21 (continued)

(b) Find during which year the population of town H first exceeds the population of town G. **3**

$P_H > P_G$

$28000e^{0.0929t} > 40000e^{0.0318t}$

$\therefore 7e^{0.0929t} > 10e^{0.0318t}$

$\therefore \frac{e^{0.0929t}}{e^{0.0318t}} > \frac{10}{7}$

$\therefore e^{0.0929t-0.0318t} > \frac{10}{7}$

$\therefore \ln e^{0.0611t} > \ln\frac{10}{7}$

$\therefore 0.0611t > \ln\frac{10}{7}$

$\therefore t > \frac{\ln\frac{10}{7}}{0.0611} \approx 5.84$ (5.83 *if students used the exact values of k_1 and k_2*)

Therefore, during 2015 or during the 6th year or during the year 6 or in the 6 years, the population of town H first exceeds the population of town G.

Comparing to the other questions in Part C, this question was poorly done.

Marks have been deducted if:

- *Students incorrectly calculated the value of t. For example: $t < 5.84$*
- *Students rounded up to have $t = 6$ and have 2016 as the answer.*
  The question asks about during which year, the beginning of 2016 is the beginning of the 7th year. See below:

2010	2011	2012	2013	2014	2015	2016
1st year	2nd year	3rd year	4th year	5th year	6th year	7th year

Question 22 (3 marks)

The share price (\$$P$) of a renewable energy company is increasing such that **3**

$$\frac{dP}{dt} = \frac{3}{t+1},$$

where t is the time in months.

If the initial share price is \$2.00, find the price after 6 months.
Write your answer to the nearest cent.

$P(t) = 3\ln(t+1) + c$

$P(0) = 3\ln(0+1) + c = 2$

$3 \times 0 + c = 2$

$\therefore\ P(t) = 3\ln(t+1) + 2$

$P(6) = 3\ln(6+1) + 2 \approx 7.84$

Generally well done.

Common mistakes:

- *Could not correctly find P(t), e.g.* $P(t) = \frac{ln(t+1)}{3} + c$, $P(t) = 3(t+1)^{-4} + c$....
- *Could not correctly calculate the value of c, e.g.* $c = \frac{2}{3}, 0$ ....
- *Careless calculation mistakes*

End of Part C Solutions

NESA Number											

YEAR 12 Mathematics Advanced

Cohort Task #4 (THSC)

Part D

Section II

Part D **16 marks**
Attempt Questions 23–26

PART D SOLUTIONS

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 23 (3 marks)

The seats in a theatre are numbered in numerical order from the first row to the last row. The first row has 14 seats. Each successive row has 4 more seats than the previous one.

(a) How many seats are there in the fourth row? **1**

$a = 14, \; d = 4$ $\frac{1}{2}$ mark

$T_4 = 14 + (4-1)4$ Generally well done.

$= 14 + 12$

$= 26$ seats $\frac{1}{2}$ mark

This means $S_n = 1320$

(b) The theatre has a maximum capacity of 1320 seats, what is the greatest number of rows of seats in the theatre? **2**

$S_n \leq 1320$

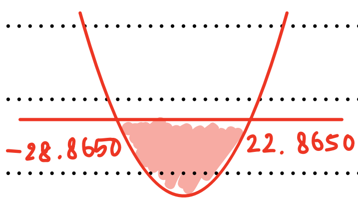
$\frac{n}{2}[2(14) + (n-1)4] \leq 1320$ 1 mark

$14n + 2n(n-1) \leq 1320$

$7n + n(n-1) \leq 660$

$n^2 + 7n - n \leq 660$

$n^2 + 6n - 660 \leq 0$



$n = \frac{-6 \pm \sqrt{6^2 + 4(6)(660)}}{2}$

$= -28.8650, \; 22.8650$ (4dp)

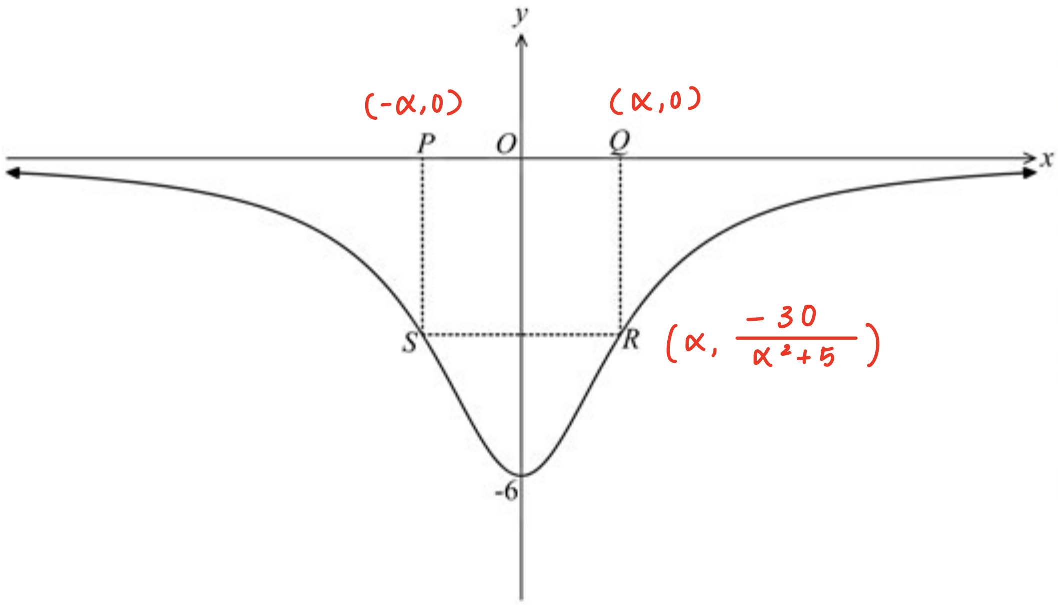
$\therefore -28.8650 \leq n \leq 22.8650$

$\therefore$ 22 full rows. 1 mark

Common errors:

- Solving for $T_n \leq 1320$
- Rounding up instead of rounding down. Round down because there isn't enough seats for a full 23rd row.

Question 24 (5 marks)



In the diagram above, the curve $y = -\dfrac{30}{x^2+5}$ has a minimum turning point at (0, –6).

A rectangle $PQRS$ is inscribed within the curve as shown with its axis of symmetry $x = 0$. = even symmetry

Q has coordinates $(\alpha, 0)$.

(a) Show that the area A, of the rectangle $PQRS$ is $A = \dfrac{60\alpha}{\alpha^2+5}$. **2**

$\because Q(\alpha, 0)$, by symmetry $P(-\alpha, 0)$

To find R, at $x = \alpha$, $y = \dfrac{-30}{\alpha^2+5}$

$QR = |y| = \dfrac{30}{\alpha^2+5}$ (length > 0) 1 mark

Clear explanation of why negative sign is omitted is required.

$A_{PQRS} = PQ \times QR$

$= (\alpha + \alpha) \times \left(\dfrac{30}{\alpha^2+5}\right)$

$= 2\alpha \cdot \left(\dfrac{30}{\alpha^2+5}\right)$

$= \dfrac{60\alpha}{\alpha^2+5}$ 1 mark

Question 24 continues on page 27

(b) Show that the maximum area of the rectangle *PQRS* is $6\sqrt{5}$ square units. **3**

$$\frac{dA}{d\alpha} = 60\left[\frac{\alpha^2+5-\alpha\cdot 2\alpha}{(\alpha^2+5)^2}\right]$$ 1 mark

$$= 60\left[\frac{5-\alpha^2}{(\alpha^2+5)^2}\right] = 0$$

$$5-\alpha^2 = 0$$

$$\alpha = \pm\sqrt{5}$$

some students did not include $\pm$ when taking square root.

$\because Q(\alpha, 0)$ and Q is on the positive side of the x-axis

$\therefore \alpha > 0$, so $\alpha = \sqrt{5}$

Some students showed $\alpha = -\sqrt{5}$ is a minimum, time could have been saved if you justified why $\alpha > 0$.

To show maximum,

α	2	$\sqrt{5}$	3
$\frac{dA}{d\alpha}$	$\frac{20}{27}$	0	$\frac{-60}{49}$
	/	—	\

Numbers required in the table.

$\therefore$ Area is maximised when $\alpha = \sqrt{5}$. 1 mark

$$A = \frac{60\sqrt{5}}{5+5}$$

$$= \frac{60\sqrt{5}}{10}$$

$$= 6\sqrt{5}\ u^2$$

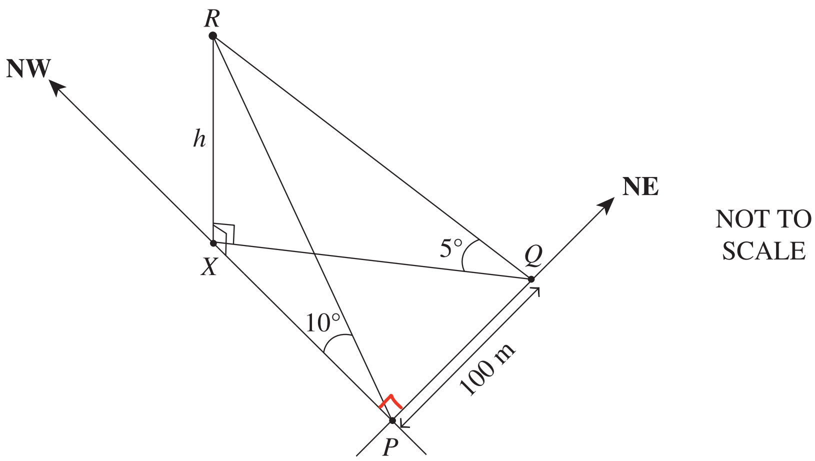
As this is a SHOW question, clear substitution is required.

$\therefore$ The maximum area is $6\sqrt{5}\ u^2$. 1 mark

End of Question 24

Question 25 (4 marks)

A hiker starts from point P and walks 100 metres along a straight path to the north-east, arriving at point Q.
A peak is located at point R, which is h metres directly above point X.
From point P the angle of elevation to peak R is 10°. At point Q, the angle of elevation to the peak is 5°.



(a) Show that $XP = h \cot 10°$, and write down a similar expression for XQ. **1**

In $\triangle RXP$, $\tan 10° = \dfrac{h}{XP}$

$XP = \dfrac{h}{\tan 10°}$

$= h\cot 10°$ $\frac{1}{2}$ mark

Similarly, $XQ = h\cot 5°$ $\frac{1}{2}$ mark

Many students did not answer the second part of the question. Make sure you read the question!!!!

(b) Hence, find the value of h. Give your answer correct to the nearest metre. **3**

$\angle XPQ = 90°$ (NW $\perp$ NE)

$XP^2 + PQ^2 = XQ^2$ 1 mark

$(h\cot 10°)^2 + 100^2 = (h\cot 5°)^2$

$h^2\cot^2 10° + 100^2 = h^2\cot^2 5°$

$h^2(\cot^2 5° - \cot^2 10°) = 100^2$

$h^2 = \dfrac{100^2}{\cot^2 5° - \cot^2 10°}$

$h = \dfrac{100}{\sqrt{\cot^2 5° - \cot^2 10°}}$ 1 mark

$= 10$ m (nearest metre) 1 mark

Some students ignored the diagram and did not see NE and NW and so $\angle XPQ = 90°$. Not realising this meant you could not answer the question successfully.

Question 26 (4 marks)

A particle moves from rest at a point A in a straight line, so that t seconds after leaving A its displacement, x metres, is given by $x = 16t^2 - 2t^4$.

Find

some students solved for $x = 0$.

(a) the time(s) for the particle to come to rest again. **2**

$v = \frac{dx}{dt} = 32t - 8t^3$ 1 mark

Particle comes to rest when $v = 32t - 8t^3 = 0$

$8t(4 - t^2) = 0$

$t = 0, \pm 2$

$t = 0$ shows when it is first at rest, and time cannot be negative.

$\therefore$ At $t = 2$ seconds, the particle comes to rest again. 1 mark

(b) the distance travelled in the first three seconds. **2**

$\because$ Particle comes to rest at $t = 2$, consider distance travelled for

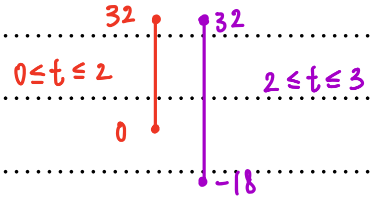
① $0 \le t \le 2$ and ② $2 \le t \le 3$.

At $t = 0$, $x = 0$

$t = 2$, $x = 16(2)^2 - 2(2)^4 = 32$

$t = 3$, $x = 16(3)^2 - 2(3)^4 = -18$

1 mark



$\therefore$ Distance travelled: $\underbrace{32}_{0 \le t \le 2} + \underbrace{(32 + 18)}_{2 \le t \le 3} = 82\text{ m}$ 1 mark

Common errors:

- Many students found at $t = 3$, $x = -18$ (displacement), and took the absolute value thinking it would be the distance travelled.

End of Part D

- Some students found x at $t = 0, 2, 3$ and did $32 + 32 - 18 = 46\text{m}$.

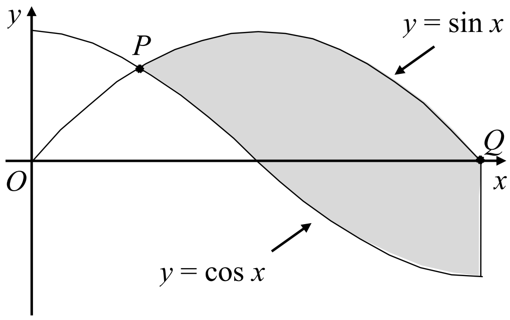
Use this space to re-write any questions for Part D.

2023 HSC MA Task 4 Part E – Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

Question 27

The graph below shows the curves $y = \sin x$ and $y = \cos x$.



A. Find the x coordinates of the points P and Q. 2

Solution	Comment(s)
At point Q, $\sin x = 0$ for the second time. This occurs at $x_Q = \pi$. At point P, $\sin x = \cos x$ for the first time. Hence: $$\begin{aligned}\sin x_P &= \cos x_P\\ \tan x_P &= 1\\ x_P &= \frac{\pi}{4}\end{aligned}$$	No half marks. Some students wasted time giving the y coordinate of P and Q when it wasn't necessary. Common error(s): • Incorrectly deducing that $x_Q = \dfrac{5\pi}{4}$.

B. Find the area of the shaded region. 3

Solution	Comment(s)
From the graph, $\sin x > \cos x$ for $\left[\dfrac{\pi}{4}, \pi\right]$. Hence: $$\begin{aligned}A &= \int_{\frac{\pi}{4}}^{\pi} \sin x - \cos x \, dx\\ &= [-\cos x - \sin x]_{\frac{\pi}{4}}^{\pi}\\ &= (-\cos\pi - \sin\pi) - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)\\ &= (1 - 0) - \left(\frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)\\ &= 1 + \sqrt{2}\end{aligned}$$	Some students chose to solve an equivalent, but more complicated integral, such as the one shown below: $$\begin{aligned}A = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x - \cos x \, dx + \int_{\frac{\pi}{2}}^{\pi} \sin x \, dx\\ + \left\lvert \int_{\frac{\pi}{2}}^{\pi} \cos x \, dx \right\rvert\end{aligned}$$ These students risked being penalised by introducing unnecessary extra steps and, consequently, more room for errors in their working. Common error(s): • Incorrectly integrating $-\cos x$ to $\sin x$.

<u>2023 HSC MA Task 4 Part E – Solutions and Comments</u>

Answers in pencil and/or with white out will **NOT** have their marks changed.

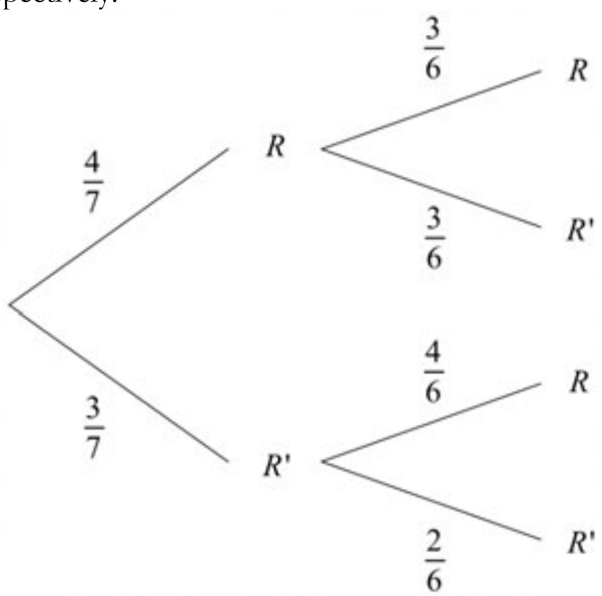
Question 28

A bag contains four red balls, two black balls and one white ball.

Wyndham selects one ball from the bag and keeps it hidden.

He then selects a second ball, also keeping it hidden.

A. Find the probability that both the selected balls are red. 1

Solution	**Comment(s)**
In the tree diagram below, R and R' are the event of selecting a red ball and not selecting a red ball respectively:  Hence: $P(RR) = \frac{4}{7} \times \frac{3}{6}$ $= \frac{2}{7}$	No half marks. Students were not required to draw a tree diagram or equivalent to score the full mark. However, given Part B, students should consider drawing one anyway.

Answers in pencil and/or with white out will **NOT** have their marks changed.

Question 28 (continued)

B. Wyndham drops one of the selected balls and we can see that it is red. 2

What is the probability that the ball that is still hidden is also red?

Solution	Comment(s)
Let X be the number of red balls in Wyndham's possession. Then: $$P(X=2 \mid X \geq 1) = \frac{P(X=2 \cap X \geq 1)}{P(X \geq 1)}$$ Since the intersection of having two red balls and having at least one red ball is just having two red balls: $$P(X=2 \mid X \geq 1) = \frac{P(X=2)}{P(X \geq 1)}$$ Furthermore, having at least one red ball is equivalent to the complement of having no red balls, so: $$\begin{aligned} P(X=2 \mid X \geq 1) &= \frac{P(X=2)}{1-P(X=0)} \\ &= \frac{P(RR)}{1-P(R'R')} \\ &= \frac{\frac{2}{7}}{1-\left(\frac{3}{7}\times\frac{2}{6}\right)} \\ &= \frac{1}{3} \end{aligned}$$	Most students received only 1 mark due to incorrectly deducing that $P=\frac{1}{2}$, with some receiving 0 from their lack of working to justify their incorrect answer. There was no information indicating that the first ball dropped was the first ball drawn, so it is incorrect to disregard the $R'R$ case.

2023 HSC MA Task 4 Part E – Solutions and Comments

Answers in pencil and/or with white out will **NOT** have their marks changed.

Question 29

On a given day, the depth of water in a river is modelled by the function:

$$h(t) = 5 + 3\sin\frac{\pi t}{4}$$

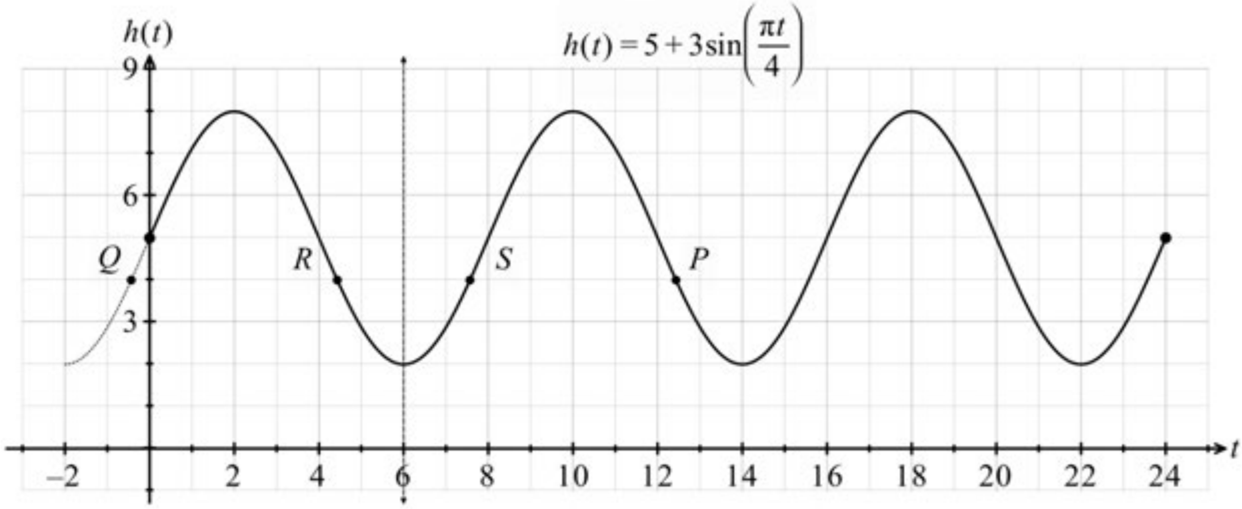
where h is the depth of the water in metres and t is the time, in hours, after 12 midnight.

A. What is the depth of the water at 12 midnight? 1

Solution	**Comment(s)**
At 12 midnight, $t = 0$. Hence: $h(0) = 5 + \sin 0$ $= 5$	No half marks. Students generally performed well on this question.

B. Sketch the graph of $h(t) = 5 + 3\sin\dfrac{\pi t}{4}$ in the domain $[0, 24]$. 3

Solution



Comment(s)

The line $x = 6$ and points P, Q, R and S pertain to Part C and are **not** relevant to the marking of Part B.

Common error(s):

- Not producing a trig curve with a period of 8 units.
- Producing a cosine curve instead of a sine curve.

Answers in pencil and/or with white out will **NOT** have their marks changed.

Question 29 (continued)

C. A family decides to go on a picnic by the river from 12 midday to 2 p.m. 3

It is only safe to swim in the river if the depth of water is less than 4 metres.

When is the earliest time the family should swim after 12 midday?

Give your answer correct to the nearest minute.

Solution	**Alternate Solution**
The point P on the graph represents the desired solution. Let $h(t) = 4$ and solve for $0 < t < 8$: $$4 = 5 + 3\sin\frac{\pi t}{4}$$ $$\sin\frac{\pi t}{4} = \frac{-1}{3}$$ $$\frac{\pi t}{4} = \sin^{-1}\frac{-1}{3}$$ $$t = \frac{4}{\pi}\sin^{-1}\frac{-1}{3}$$ $$= -0.433 \text{ (3 d.p.)}$$ Recognise from the graph in Part B that: • $t_Q = \frac{4}{\pi}\sin^{-1}\frac{-1}{3}$ • P and Q are symmetrical about $x = 6$. Hence: $$t_P = 6 + (6 - t_Q)$$ $$= 6 + \left(6 - \frac{4}{\pi}\sin^{-1}\frac{-1}{3}\right)$$ $$= 12{:}26 \text{ p.m. (n. min)}$$	Solving for the related angle: $$\sin\frac{\pi t}{4} = \frac{1}{3}$$ $$\frac{\pi t}{4} = \sin^{-1}\frac{1}{3}$$ As $\sin\frac{\pi t}{4} < 0$: $$\frac{\pi t}{4} = \pi + \sin^{-1}\frac{1}{3} \qquad \frac{\pi t}{4} = 2\pi - \sin^{-1}\frac{1}{3}$$ $$t = \frac{4}{\pi}\left(\pi + \sin^{-1}\frac{1}{3}\right) \qquad t = \frac{4}{\pi}\left(2\pi - \sin^{-1}\frac{1}{3}\right)$$ $$= 4 + \frac{4}{\pi}\sin^{-1}\frac{1}{3} \qquad = 8 - \frac{4}{\pi}\sin^{-1}\frac{1}{3}$$ $$= 4.433 \text{ (3 d.p.)} \qquad = 7.567 \text{ (3. d.p.)}$$ From the graph: $t_R = 4.433$ $\qquad$ $t_S = 7.567$ Since $h(t)$ has a period of 8 hours: $$t_P = 8 + t_R$$ $$= 8 + \left(4 + \frac{4}{\pi}\sin^{-1}\frac{1}{3}\right)$$ $$= 12{:}26 \text{ p.m. (n. min)}$$

Comment(s)
Most students could only score a maximum of 2 marks for providing insufficient or incorrect working in subsequently finding the correct time of 12:26 p.m. Students who deduced that $\frac{\pi t}{4} = \sin^{-1}\frac{-1}{3}$ were penalised if they failed to explicitly state that this time corresponds to point Q on the graph in Part B. Meanwhile, students who deduced that $\frac{\pi t}{4} = \sin^{-1}\frac{1}{3}$ were penalised if they failed to explicitly state that this is the related angle and **not** due to time being positive. In the context of this question, "negative" time simply refers to time before midnight, which is perfectly valid. Common error(s): • Not using radians when solving for time, which is not measured in degrees.

Part F Solutions (14 marks)

Question 30 (2 marks)

Evaluate $\lim_{x\to 4}\frac{x^2-2x-8}{\sqrt{x}-2}$. **2**

Show all working.

Method 1

$$\lim_{x\to 4}\frac{x^2-2x-8}{\sqrt{x}-2}=\lim_{x\to 4}\frac{(x-4)(x+2)}{\sqrt{x}-2}\times\frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$=\lim_{x\to 4}\frac{(x-4)(x+2)\left(\sqrt{x}+2\right)}{(x-4)}$$

$$=\lim_{x\to 4}(x+2)\left(\sqrt{x}+2\right)$$

$$=24$$

Method 2

$$\lim_{x\to 4}\frac{x^2-2x-8}{\sqrt{x}-2}=\lim_{x\to 4}\frac{(x-4)(x+2)}{\sqrt{x}-2}$$

$$=\lim_{x\to 4}\frac{\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)(x+2)}{\sqrt{x}-2}$$

$$=\lim_{x\to 4}(x+2)\left(\sqrt{x}+2\right)$$

$$=24$$

Comment

Use of notation was not done well. Plus too many students think that $\frac{0}{0}=0$!!!

Also, there are some students who think that $\sqrt{4}=\pm 2$. I thought this thinking had finished in Y7.

If students want to use non-syllabus theorems, like L'Hopital's Rule, they need to justify why the theorem works.

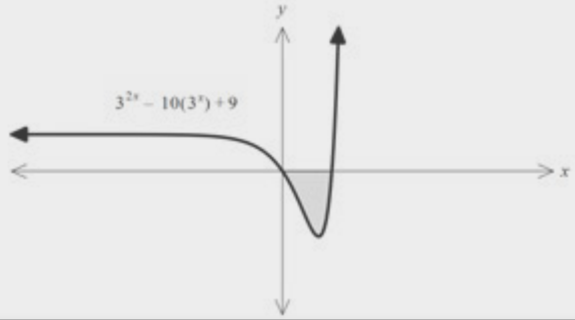
Students who just used approximations to get the final answer did not get full marks. There was no justification why their answer was correct as they did not show ALL necessary working.

Part F Solutions (14 marks) (continued)

Question 31 (5 marks)

The graph of $f(x)=3^{2x}-10\left(3^x\right)+9$ is shown below. **5**

Find the exact value of the area shaded.



$$f(x)=(3^x)^2-10(3^x)+9$$
$$=(3^x-1)(3^x-9)$$

$\therefore$ x-intercepts are $x = 0, 2$

$$\text{Shaded area} = \left|\int_0^2 f(x)\ dx\right| \text{ u}^2$$

$$\int_0^2 f(x)\ dx = \int_0^2 3^{2x}-10(3^x)+9\ dx$$

$$=\left[\frac{1}{2\ln 3}3^{2x}-\frac{10}{\ln 3}3^x+9x\right]_0^2$$

$$=\left(\frac{1}{2\ln 3}\times 81-\frac{10}{\ln 3}\times 9+9\times 2\right)-\left(\frac{1}{2\ln 3}-\frac{10}{\ln 3}+0\right)$$

$$=\frac{81-1-180+20}{2\ln 3}+18=18-\frac{80}{2\ln 3}$$

$$=18-\frac{40}{\ln 3}$$

$$\therefore \text{area} = \left(\frac{40}{\ln 3}-18\right)\text{ u}^2.$$

Comment

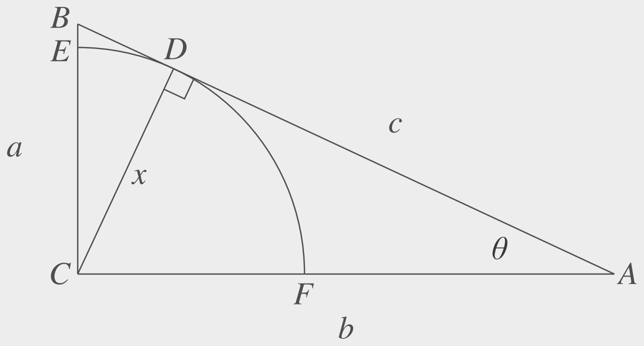
Many students did not observe that the integral must be negative (looking at the graph), and so did not take the absolute value of their answer. This became problematic for the students who didn't integrate properly and came up with a positive answer – this should have been a check that a mistake was made!
If students didn't show recognition that the shaded area would produce a 'negative' area, then they could not get full marks.

The final answer had to be somewhat simplified to ensure full marks.

Students who tried integration by substitution did not know what they were doing, but they were not going to get many marks, as their working became basically rubbish and so not worth anything.

Many students think that $|a-b|=a+b$.

Question 32 (7 marks)
In triangle ABC, BC is perpendicular to AC.
Side BC has length a, side AC has length b and side AB has length c.
A quadrant of a circle of radius x, centred at C, is constructed.
The arc meets side BC at E. It touches the side AB at D, and meets side AC at F.
The interval CD is perpendicular to AB.



(a) Explain why $x = \dfrac{ab}{c}$ **2**

Method 1: Trigonometry

Let $\angle CAB = \theta$

In ΔABC: $\sin\theta = \dfrac{a}{c}$

In ΔACD: $\sin\theta = \dfrac{x}{b}$

$\therefore \dfrac{a}{c} = \dfrac{x}{b} \Rightarrow x = \dfrac{ab}{c}$

Method 2: Similarity

$\Delta ABC \,|||\, \Delta ACD$ (AA)

$\therefore \dfrac{x}{a} = \dfrac{b}{c}$ (matching sides of similar triangles)

$\therefore x = \dfrac{ab}{c}$

Comment

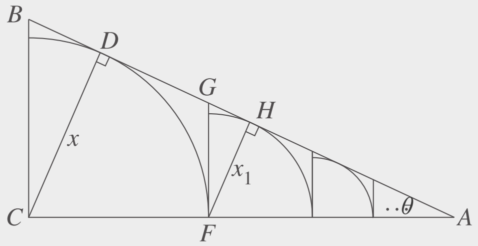
Some students used area to get the right solution.
Students who used the similarity method had to indicate which triangles were similar and why.
Students did not need to do a full similarity proof, but they did have to indicate that the ratio of matching sides is preserved with similar triangles.
It was also very unlikely that students who used congruent triangles could get full marks.

Some students didn't use the fact that this question was worth 2 marks to help gauge the amount of working needed.

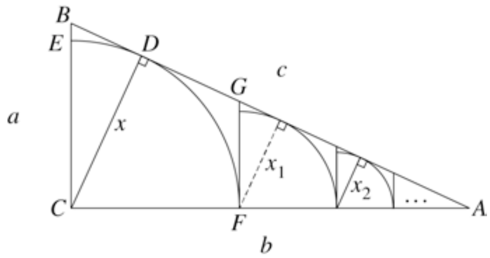
Too many students confused DC with c.
Even though $\Delta DBC \,|||\, \Delta DCA$, it didn't help with the solution.

Question 32 (continued)

From F, a line perpendicular to AC is drawn to meet AB at G, forming the right-angled triangle GFA.
A new quadrant is constructed in triangle GFA touching side AB at H.
The process is then repeated indefinitely.



(b) Show that the limiting sum of the areas of all the quadrants is $\dfrac{\pi ab^2}{4(2c-a)}$. **4**



$$\text{Area} = \frac{1}{4}\pi x^2 + \frac{1}{4}\pi x_1^{\,2} + \frac{1}{4}\pi x_2^{\,2} + \ldots = \frac{1}{4}\pi\left(x^2 + x_1^{\,2} + x_2^{\,2} + \ldots\right)$$

Method 1: Trigonometry

Using part (a):

$$\sin\theta = \frac{x}{b} = \frac{x_1}{b-x} \Rightarrow \frac{x_1}{x} = \frac{b-x}{b} = 1-\frac{x}{b} = 1-\frac{\frac{ab}{c}}{b} = 1-\frac{a}{c}$$

$$\therefore x_1 = x\left(1-\frac{a}{c}\right)$$

Similarly, $x_2 = x_1\left(1-\dfrac{a}{c}\right) = x\left(1-\dfrac{a}{c}\right)^2$ and so on.

$\therefore$ Area $= \dfrac{1}{4}\pi x^2\left[1+r^2+r^4+\ldots\right]$, where $r = 1-\dfrac{a}{c}$.

Note: As c is the hypotenuse of $\Delta\, ABC$ then $0<\dfrac{a}{c}<1$ and so $0<1-\dfrac{a}{c}<1$.
$\therefore 0<r^2<1$

$\therefore$ a limiting sum exists.

Method 1 is continues over the page

Question 32 (b) (continued)

Method 1: Trigonometry (continued)

$$S_{\infty} = \frac{A}{1-r} \qquad \left[A = \frac{1}{4}\pi x^2\right]$$

$$= \frac{1}{4}\pi x^2 \times \frac{1}{1-\left(1-\frac{a}{c}\right)^2}$$

$$= \frac{1}{4}\pi \frac{a^2b^2}{c^2} \times \frac{1}{\frac{2a}{c}-\frac{a^2}{c^2}} \times \frac{c^2}{c^2}$$

$$= \frac{1}{4}\pi a^2b^2 \times \frac{1}{2ac-a^2}$$

$$= \frac{1}{4}\pi a^2b^2 \times \frac{1}{a(2c-a)}$$

$$= \frac{\pi ab^2}{4(2c-a)}$$

Method 2 is over the page

Method 2: Similarity

Consider $\Delta\, GFA$: $AF = b - x$

Since $\triangle ABC$ and $\triangle AGF$ are right-angled and angle A is common, then $\triangle ABC \,|||\, \triangle AGF$

$$\frac{GF}{a} = \frac{b-x}{b}$$
$$\frac{GF}{a} = \frac{1}{b}\left(b - \frac{ab}{c}\right)$$
$$\frac{GF}{a} = \frac{b}{b}\left(1 - \frac{a}{c}\right)$$

It follows then

$$\frac{x_1}{x} = 1 - \frac{a}{c}$$
$$x_1 = \left(1 - \frac{a}{c}\right)x$$
$$x_1 = (1 - \sin A)\,x$$

The ratio of the lengths of the radii are independent of a, b and c.

Hence, successive radii will be in the ratio $(1 - \sin A)$.

The rest of the proof continues as in Method 1

Comment

Some students were able to get marks here, though the setting out in general was deplorable.
Some students were penalised for this.

It seemed that there was a great need to fudge the answer, rather than go back to previous sections and check those.

Bad setting out didn't help.

Question 32 (continued)

(c) Hence, or otherwise, show that $\frac{\pi}{2} < \frac{2c-a}{b}$. **1**

The area of the sum of quadrants, found in part (b), is smaller than the area of the triangle ABC

$$\therefore \frac{ab}{2} > \frac{\pi ab^2}{4(2c-a)}$$

$$\therefore \frac{\pi}{2} < \frac{2c-a}{b}$$

Comment

Students had to explain why $\frac{ab}{2} > \frac{\pi ab^2}{4(2c-a)}$ was true in the first place.

Then, there had to be some working from the statement $\frac{ab}{2} > \frac{\pi ab^2}{4(2c-a)}$ to the required statement.

For example:

$$\frac{\pi ab^2}{4(2c-a)} < \frac{ab}{2}$$

$$\frac{\pi}{4} < \frac{ab}{2} \times \frac{(2c-a)}{ab^2}$$

$$\frac{\pi}{4} < \frac{2c-a}{2b}$$

$$\frac{\pi}{2} < \frac{2c-a}{b}$$

End of solutions